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CRACK INITIATION ANGLE UNDER UNIAXIAL AND BIAXIAL LOADINGS IN HIGH DENSITY POLYETHYLENE

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In this work, a prediction of crack initiation under uniaxial and biaxial loading is presented. Experimental tests and finite element analyses were performed on high-density polyethylene (HDPE) center-crack tension (CCT) specimens. Under uniaxial loading, the crack initiation angle was evaluated experimentally and calculated numerically using the strain energy density (SED) approach. The results showed that under uniaxial loading, the crack initiation direction is perpendicular to the direction of the applied load, i.e. the direction of crack growth takes the direction corresponding to the minimum SED values. Under biaxial loading, the initiation angle is estimated for different values of the biaxial ratio R and for different values of the crack inclination; in this case, the crack tends to propagate in the direction perpendicular to the most dominant loading direction.

Keywords: Crack direction; uniaxial loading; biaxial loading; Strain energy density; HDPE; Mixed mode.

1. Introduction

Nowadays, the development of fluid transportation's techniques requires a wide integration of new materials into the pipelines manufacturing domain for gas and drinking water transmission networks, such as High Density Polyethylene (HDPE). The use of this material provides several advantages thanks to the physicochemical and mechanical characteristics. From mechanical point of view, the reliability of this material depends strongly on its resistance to several phenomena, such as cracking and failure. Therefore, researchers in the domain of fracture mechanics interest in initiation and crack propagation phenomenon. In the presence of a pre-crack, the prediction of the angle of initiation in structures is introduced among the major objectives in the study of the propagation. This objective was not easily accessible, especially for nonlinear materials and under

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multiaxial loading. Nevertheless, the prediction of the initial angle of propagation becomes possible through the development of the Finite Element Method (FEM).

Stress concentrations often lead to crack initiation, which is unpredictable and difficult to model. Indeed, the numerical studies carrying out on the influence of the position and the initiation angle around the defects and their mode (pure or mixed mode) [1]. Several fracture criteria for cracks subjected to mixed mode loading have been introduced to estimate the crack initiation angle.

In Linear Elastic Fracture Mechanics (LEFM), many technics are using in the field of crack initiation prediction of the local stress and displacement fields at the crack tip as Stress intensity Factor (SIF) K, Maximum strain criterion [2, 3], the criterion of maximum principal tangential stress [4] and the criterion of maximum tangential stress [5]. In nonlinear materials, J-integral is used as a measure of the intensity of deformation at the notch or crack tip. [6]. J-integral is primarily used for uniaxial loading conditions and may not accurately express the effects of biaxial loading [7].

Prediction of crack initiation and its angle is made more difficult by the complex stress state and the interaction between in-plane and out-of-plane stresses. For this reason, the choice of an appropriate criterion for crack growth is very important in the field of fracture mechanics [8].

Strain Energy Eensity criterion (SED) can handle complex stress states and mixed-mode loading conditions [9-11]. Strain Energy Density criterion of Sih [9, 10, 12] based on evaluation of strain energy density values of elements surrounding the crack tip and the strain energy density factor (S) as an indicator parameter. The initial angle of crack growth at the crack tip is assumed to take a radial direction along the element with the smallest factor S. The application of the SED method to the field of crack initiation and propagation remains an active topic, as it is common under multiaxial loading conditions [13] and can account for both elastic and plastic deformations as evidenced by many recent studies [14-23].

This paper provide a prediction of the initial crack growth angle in high density polyethylene using the method of strain energy density in both uniaxial and biaxial mixed mode. We present in this work a finite element analysis of the crack growth direction using the local minimum value of SED around the crack tip $(dW/dV)_{min}$ and the finite element software Ansys Parametric Design Language (APDL). Experimental tests was achieved on thin plate in order to validate the results obtained by the SED criterion.

2. SED theory

Sih [10] formulates the specific factor of stain energy density *S* from the crack tip at the distance *r*, in term of the energy as follows:

$$\frac{dW}{dV} = \frac{S}{r} \qquad (1)$$

Thus:

$$S = r.\frac{dW}{dV} \qquad (2)$$

Where: $\frac{dW}{dV}$ is strain energy density function (J/mm^3).

S: Strain energy density factor (J/mm^2).

r: the distance between the element center and the crack tip (mm).

The likely direction of crack growth can be determined by estimating the smallest factor of the SED S_{min} surrounding the crack tip. The SED can be evaluates from the equation (3).

$$\frac{dW}{dV} = \int_0^{\varepsilon_{ij}} \sigma_{ij}\, d\varepsilon_{ij} \qquad (3)$$

Where and ε_{ij} is the strain component (%) and σ_{ij} is stress component (MPa).

3. Mixed mode fracture tests

Fracture tests have been performed on samples with inclined center crack with length of a=20 mm.

Specimens' dimensions are represented (Fig.1) have length of 120 mm and width of 75 mm. various pre-crack inclination have been chosen α= [0°, 30° , 45° , 60°]. specimens have been subjected under uniaxial loading under controlled force F = 10 KN and speed test v = 50 mm/min. once specimens fixed in the machine, the free dimensions are 75mm x 100 mm. Due to the crack inclination, mixed mode (I*II) can be generated around the crack-tip. Fig.1 depicts the samples' geometries used in the tests (mode I and mixed-mode).

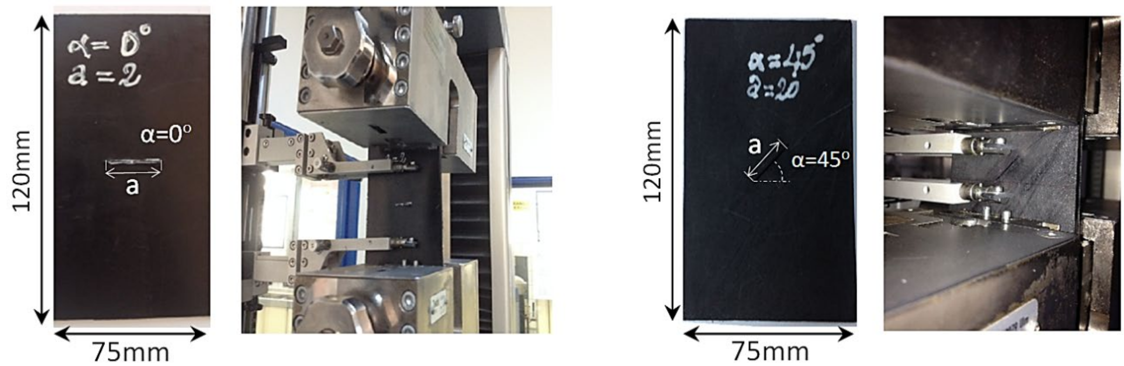
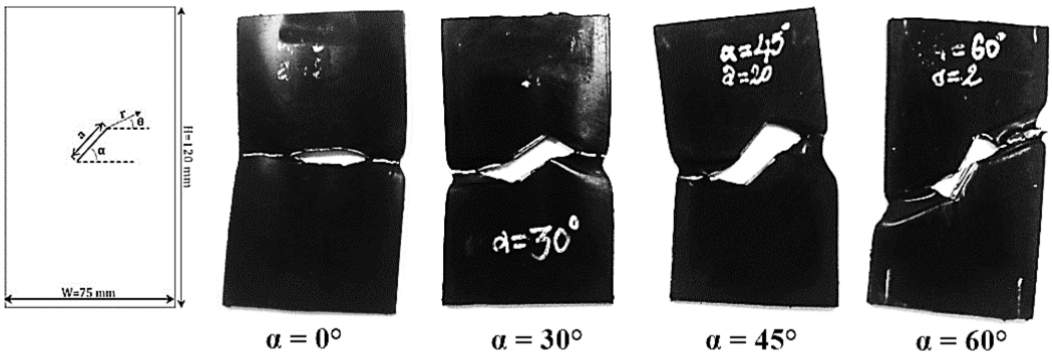


Fig. 1. CCT Geometry used in fracture tests (Center Cracked Tension specimens).

Angle of crack initiation

The angle θ_0 is the initial direction of crack growth from the pre-crack (Fig.2a–d). For different values of α, the values of θ_0 have been evaluated in each test. Figure 2b depicts the initial crack growth angle assessed in this study (Table 1).



a) b)

Fig.2. a) Geometric parameters of initiation angle. b) Specimens after fracture tests.

A sequence of images retraces the progress of fracture tests is shown in Figure 3. The observation of the specimens during the fracture tests allows us to make the following remarks:

1- Blunting of the ends of the initial crack (Fig.3.a),
2- The emergence of an almost triangular crack tip area (Fig.3.b),
3- The initiation of the crack before the ligament is completely plasticized (Fig.3.c), and development of the plasticization at the crack tip.
4- The appearance of streaks caused by the tearing of the specimen.
5- Complete plasticization of the ligament length with the formation of necking and ductile shear of the ligament (Fig.3.d).

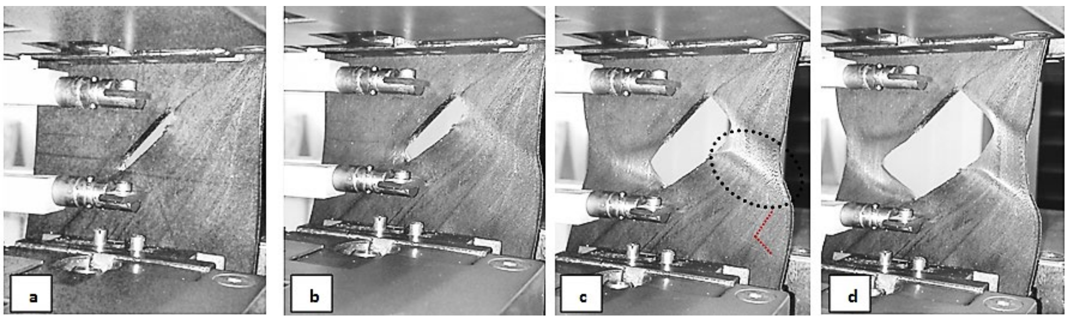


Fig. 3. Progress in fracture testing of HDPE.

Finally, It should be mentioned that these observations are noted by Hamlaoui et al. [24], using a tensile test carried out on SENT HDPE-100 specimens.

Table 1

Initiation angle values obtained by fracture tests θ_0 (°)

sample code	Crack inclination α (°)	Crack initiation θ_0 (°)
1	0	-5
2	30	-45
3	45	-64
4	60	-83

4. Finite elements study

2D finite element analysis have been carried out on CCT geometry (centre cracked tensile specimen) using Ansys Parametric Design Language (APDL). The inclination angle of the pre-crack represented by α= (0°, 30°, 45° and 60°).

To create the mesh of the fractured plate in Fig.4, element type, "PLANE183" is employed. This element type is a higher order two dimensional, eight-node element with the capacity to construct a triangular-shaped element, which is necessary at the crack tip zone, and two degrees of freedom at each node (translations in the nodal x and y directions). It also exhibits quadratic displacement behavior.

For uniaxial loading, we use 2D plate with same dimension used in experimental tests (120mm x 75mm). As boundary conditions, the lower bound of the plate is fixed. Whereas, the upper bound is loaded by a controlled displacement (d).

For biaxial study, we consider a thin square plate of dimensions (100 mm x 100 mm) to ensure that the pre-crack is located in the center of the sample. In this way, we neglect the effect of the specimen's dimensions on the load distribution at the ends of the specimen and, therefore, the effect of the crack growth direction. The plate is fixed from the lower and the left bounds (Block all degrees of freedom) and subjected to biaxial loading tension-tension with controlled displacement d_x and d_y from the upper and right bounds in both direction $\overrightarrow{OX}$ and $\overrightarrow{OY}$ respectively. Figure 4.b shows the geometric model of the pre-cracked plate subjected to biaxial tension.

Due to the singularity of the stress field near the crack, the elements near the crack tip show a strong singularity, to avoid the calculation divergence we avoid calculating the strain energy density at the singular elements. Fig.4.a, shows the mesh used for the numerical calculations. In Figure 4.a, r_i is the distance between the center of element 'c' and the crack tip.

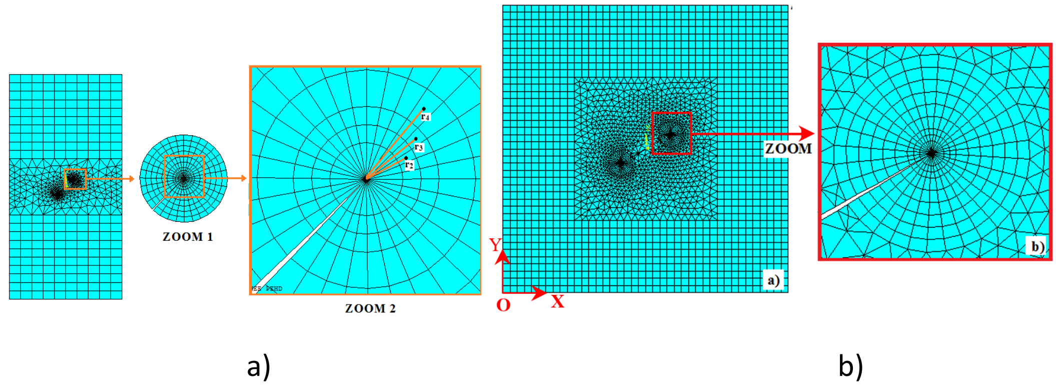


Fig.4. Finite element model used for:
a) Uniaxial loading (center cracked plate);
b) Biaxial loading (square plate with α = 45 °).

In this study, the material properties are represented by the true stress-strain curve (Fig.5) obtained by tensile test used by Berrehili [25]. According to this experimental curve, the material behavior considered as elastic-plastic with multi-linear isotropic hardening [25-27].

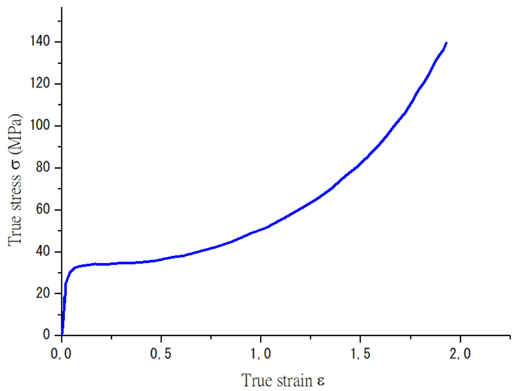


Fig.5. Material behavior of the HDPE used in the study.

5. Results and discussion
5.1. Uniaxial loading analysis
SED calculation

(Fig. 6) represents the FE analysis results recorded for the following pre-crack inclinations α = [0°, 30°, 45° and 60°], with several values of radius ri. These figures clearly show that from the central zone (in this case r≥r1) surrounding the crack tip, the minimum value of (dW/dV) is obtained for constant values whatever the radius r.

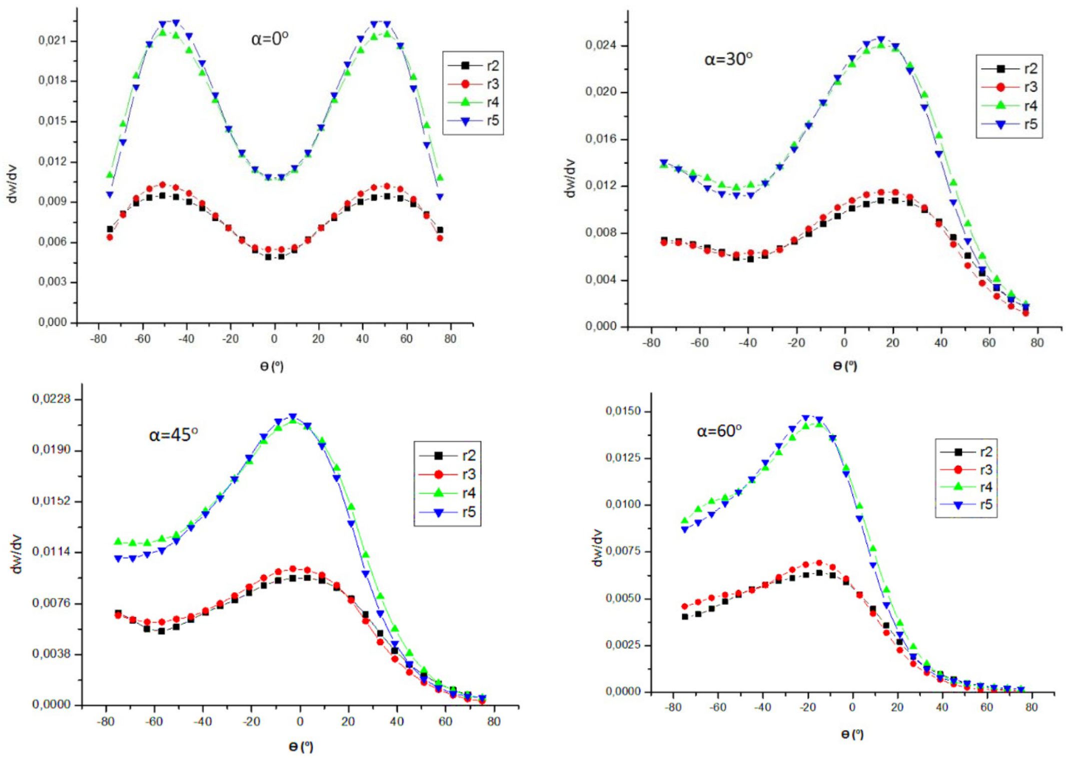


Fig.6. SED values as a function of θ for various radius (α = 0°, 30°, 45°, and 60°).

The angle θ_0 corresponds to the direction normal to the load. The results obtained numerically compared to the results predicted experimentally.

These results demonstrate good correlation between the two approaches. Fig.7 shows a comparison of the initial angle of crack growth θ_0 (°) obtained by the fracture test and the FE Analysis results using Strain energy density (SED Method).

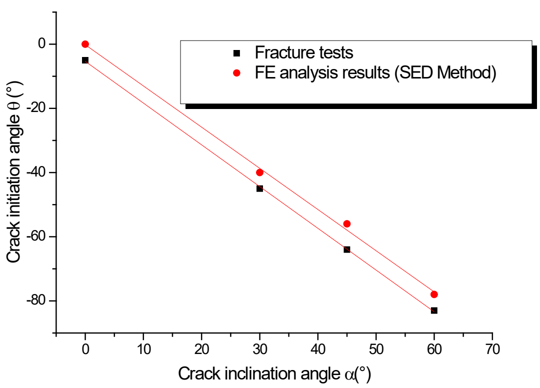


Fig.7. Comparison of initiation angle θ_0 (°) obtained by fracture test and FE results (using SED method).

5.2 Biaxial loading analysis

In this analysis, the biaxial loading is characterized by the biaxility ratio $R = \frac{d_x}{d_y}$ (with $\mathrm{d_y} = 1\mathrm{mm}$) . d_x and d_y represent the displacements along the axes (Ox) and (Oy), respectively.

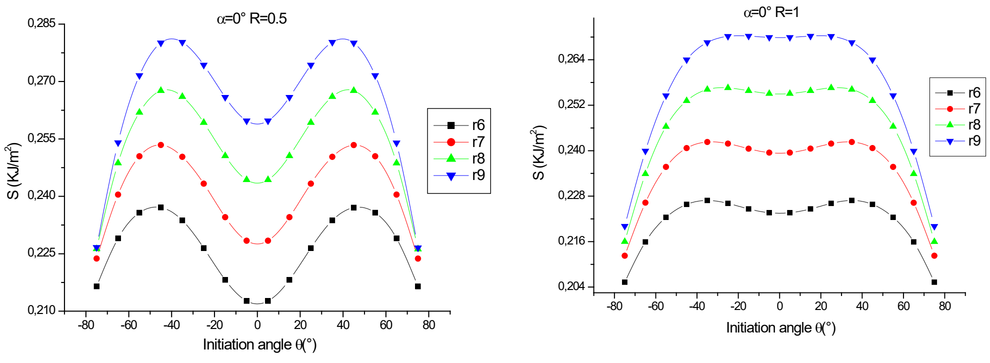
Strain energy density

We examined the variation of the strain energy density factor S as a function of the angle θ and the radius r. The curves are plotted for different ratios R (with R= 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75 and 2). The central crack is supposed to propagate in the horizontal direction (α= 45°). An example of the results obtained is given in Figure 8.

The obtained result shows that:

For a biaxiality ratio R<1 (with α= 45°, the angle θ_0 takes null values independent of the ratio R. This can be explained by the fact that the crack propagates horizontally according to the mode-I (opening mode). The direction of propagation is perpendicular to the direction of the largest load.

For $R = 1$ ($\mathrm{d_y} = \mathrm{d_x}$), the minimum of the factor S reached a value of θ_0=0°, i.e: For $R > 1$ ($\mathrm{d_x} > \mathrm{d_y}$), the probable crack growth direction is always horizontal and the plotted curves show no obvious local minimum of the factor S. In this case, the domination of loading along the (Ox) axis prevents crack opening, so no crack growth is possible.



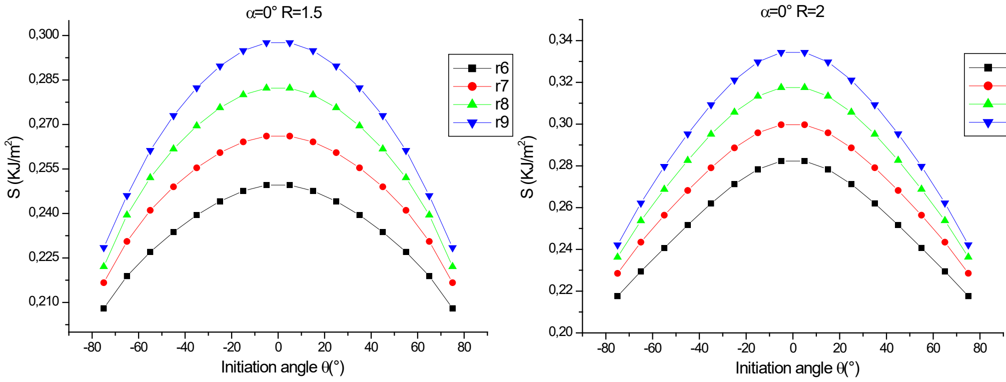


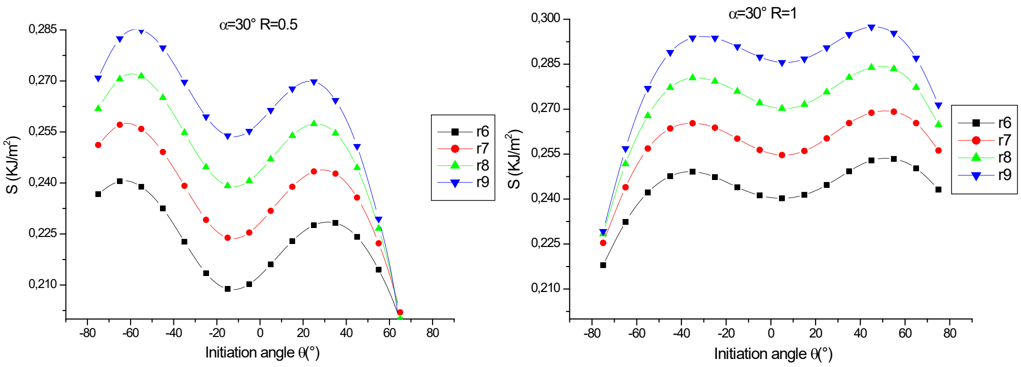
Fig.8. Effect of ratio R on the criterion (α= 0°).

In this section, we calculated the variation of the factor S for different ratio R and for different distances r measured from the crack tip. Fig.9, 10 show the evolution of S as a function of the radius r and the polar angle θ (with α= 30°and α= 45°) respectively.

The obtained result shows that:

Outside the core area around the crack tip, the minimum value of SED obtained a constant value of biaxial ratio R, i.e. Each inclined crack having a critical ratio R_{crit} in which the crack changes its direction of propagation from negative to positive one.

In the case where α= 45° (with R=1), the crack propagates along the same plane of the pre-crack, along the axis that represents the symmetry in geometry and applied loading.



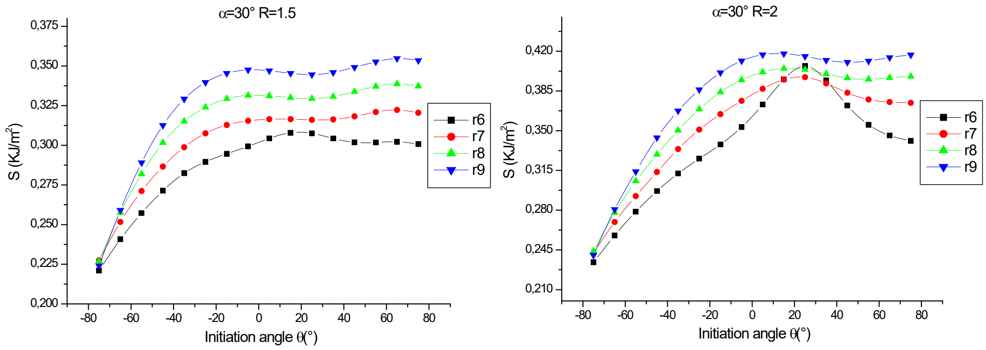
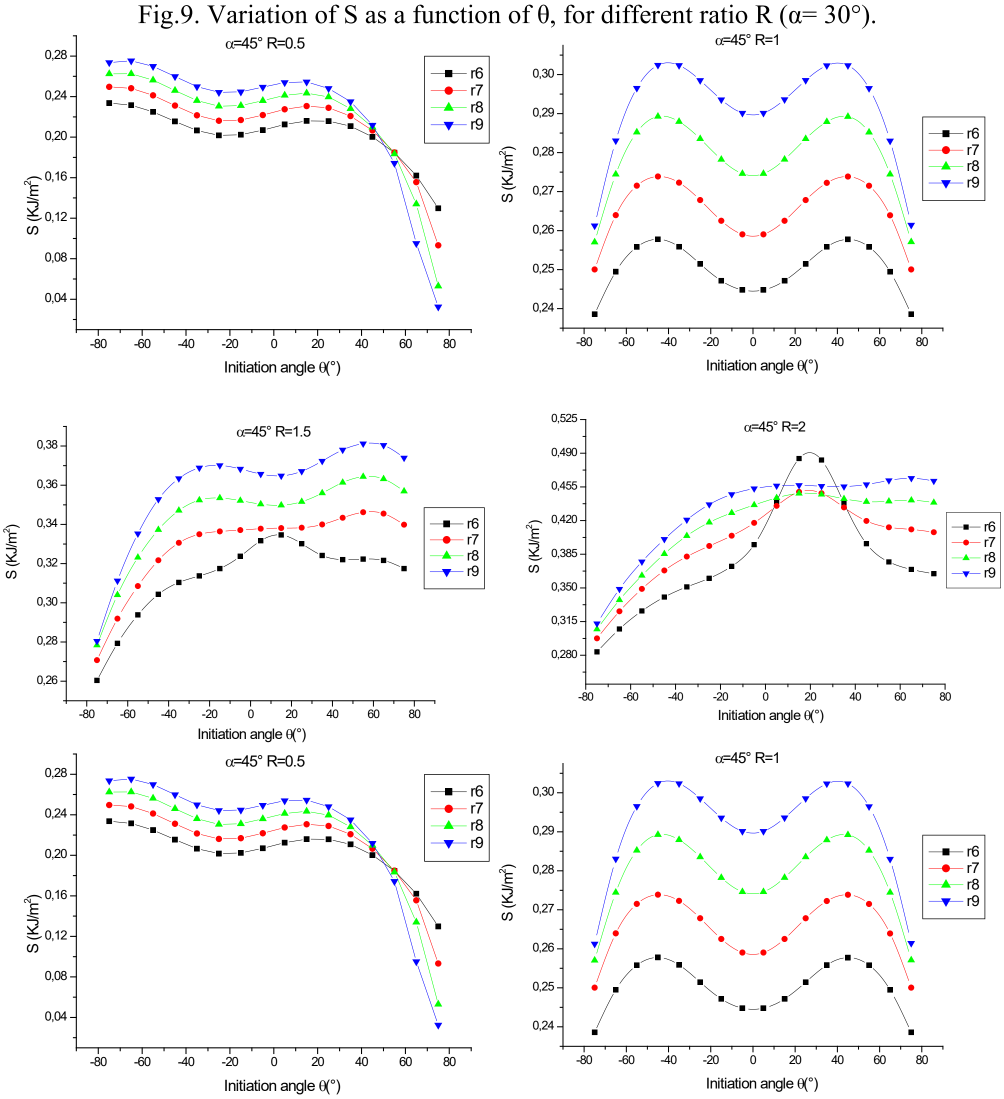


Fig.9. Variation of S as a function of θ, for different ratio R (α= 30°).



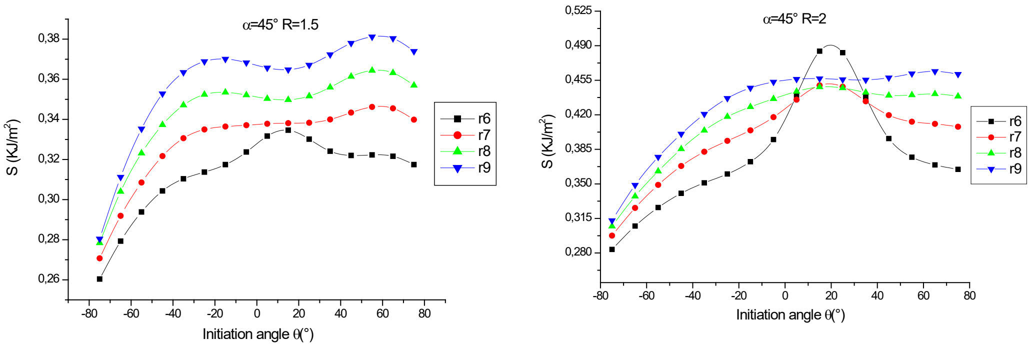


Fig.10. Variation of S as a function of θ, for different ratio R (α= 45°).

To further highlight the impact of the biaxiality ratio R on the crack direction prediction, we have depicted the variation of the bending angle as a function of the ratio R in Figure 11.The obtained results are given for four angles of inclination α (with α=0°, 30°, 45° and 60°).

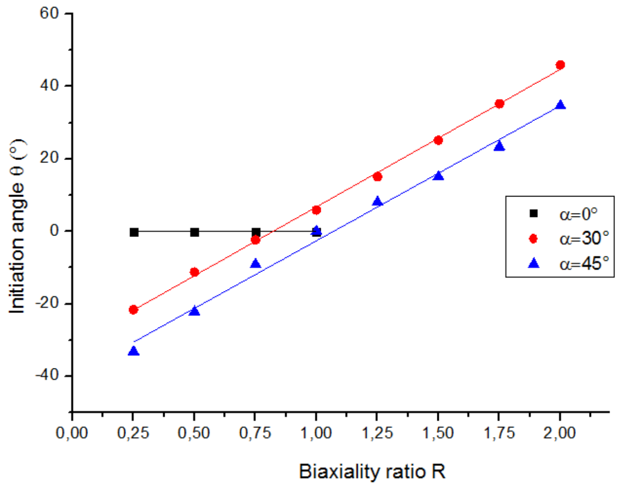


Fig. 11. Effect of biaxial ratio R on the initial angle of crack growth θ_0.

The obtained result shows that:

- For a crack supposed to initiate horizontally (α= 0°) and if $R \leq 1$, the crack will propagate according to the opening mode (mode I). In the opposite case ($R > 1$), no propagation is possible because of crack closing which caused itself by the domination of the parallel loading to the crack plane.
- Local minima of strain energy density are not clearly discernible at radius near the crack tip; which amounts to the great singularity and elements distortion at these zones, i.e. the further one moves away from the crack tip, the more

obvious the local minimum value is. That is why the values of Strain energy density should be determined from the large radius.

- For pre-cracks ($\alpha \neq 0°$), the estimate of the initial crack growth θ_0 is strongly linked to the ratio R. The evolution of the initiation angle for each pre-crack is linear proportional to the biaxility ratio R.
- The plane corresponding to the angle $\theta_0 = 0°$ represents the critical plane in which the crack changes its direction of propagation.

6. Conclusions

This study has been performed to analyze the initial crack growth angle of HDPE under uniaxial and biaxial loading of HDPE using strain energy density criterion. For different pre-crack inclination, the initiation angle is evaluated as a function of the minimum strain energy density factor (MSED) around the crack tip. The results obtained lead to the following conclusions:

Under uniaxial loading:

- Elements surrounding the crack tip have been used. This mesh allows us to determine the density of deformation energy in this zone and identify the initial angle of crack propagation in the direction at which this energy density is minimized $(dW / dV)_{min}$.
- The minimum value of SED reaches a constant value regardless of radius r. Angle θ_0 corresponds to the horizontal plane normal to the load direction. This result suggests that the estimated direction of crack propagation is always perpendicular to the loading direction.
- The value of the angle θ_0 numerically obtained by this criterion is compared with the experimentally estimated value. This comparison shows good agreement with the two approaches.

Under biaxial loading:

- The value of θ_0 of each inclined crack ($\theta \neq 0°$) is linear proportional to the biaxility ratio R.
- In the case where $\alpha = 45°$ (with R=1), the crack propagates along the same plane of the pre-crack, along the axis that represents the symmetry in the geometry and in the loading applied.

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