KINEMATICS OF THE HEXAPOD PARALLEL ROBOT

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Articolul prezintă o modelare recurentă pentru cinematica unui robot paralel Hexapod. Controlat de şase forţe, prototipul robotului paralel este un system mecanic spaţial cu şase grade de libertate şi cu şase lanţuri cinematice de lungimi constante care se conectează la platforma mobilă. Cunoscând poziţia şi mişcarea generală a platformei, se dezvoltă problema de cinematică inversă şi se determină poziţiile, vitezele şi acceleraţiile fiecărui element al robotului. În final se obţin ecuaţii matriceale compacte şi grafice de simulare pentru deplasările, vitezele şi acceleraţiile de intrare ale celor şase acţionori.

Recursive modelling for the kinematics of the Hexapod parallel robot are established in this paper. Controlled by six forces, the parallel manipulator prototype is a space six-degrees-of-freedom mechanical system with six fixed-length legs connecting to the moving platform. Knowing the position and the general motion of the platform, one develop the inverse kinematics problem and determine the position, velocity and acceleration of each manipulator element. Finally, compact matrix relations and graphs of simulation for the input displacements, velocities and accelerations of six actuators are obtained.

Keywords: Connectivity conditions; kinematics; parallel robot

1. Introduction

Parallel manipulators are closed-loop mechanisms presenting very good potential in terms of accuracy, stiffness and ability to manipulate large loads. In general, these manipulators consist of two main elements coupled through numerous legs acting in parallel. One body is arbitrarily designated as fixed and is called *base*, while the other is regarded as movable and hence is called *moving platform* of the manipulator. Several mobile legs or limbs, made up as serial robots, connect the movable platform to the fixed frame. The bodies of the robot are connected one to the other by spherical joints, universal joints, revolute joints or prismatic joints. Typically, the number of actuators is equal to the number of degrees of freedom such that every link is controlled at or near the fixed base [1].

Parallel mechanisms could be found in practical applications, where it is advisable to orient a rigid body in space of high speed, such as aircraft simulators

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[2], positional tracker and telescopes [3], [4]. The first design for industrial purposes can be dated back to 1962, when Gough implemented a six-linear jack system for use as a Universal tire-testing machine. In fact, it was a huge force sensor, capable of measuring forces and torques on a wheel in all directions. Some years later, Stewart published a design of a platform robot to be use as a flight simulator.

In comparison with serial mechanisms, parallel manipulator is a complex mechanical structure, behaving some special characteristics such as: greater stiffness, potentially higher kinematical precision, stabile capacity and suitable position of actuators arrangement. However, they suffer due to the problems of relatively small useful workspace and design difficulties.

Considerable efforts have been devoted to the kinematics and dynamic analysis of fully parallel manipulators. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [2]; Merlet [5]; Parenti Castelli and Di Gregorio [6]). They are used in flight simulators and more recently for Parallel Kinematics Machines. The prototype of Delta parallel robot (Clavel [7]; Tsai and Stamper [8]; Staicu [9]) developed by Clavel at the Federal Polytechnic Institute of Lausanne and by Tsai and Stamper at the University of Maryland as well as the Star parallel manipulator (Hervé and Sparacino [10]) are equipped with three motors which train on the mobile platform in a three-degree-of-freedom general translation motion. Angeles [11], Gosselin and Gagné [12], Wang and Gosselin [13], Staicu [14] analysed the direct kinematics, dynamics and singularity loci of the Agile Wrist spherical parallel robot with three concurrent actuators.

The analysis of parallel manipulators is usually implemented through analytical methods in classical mechanics, where projection and resolution of vector equations on the reference axes are written in a considerable number of cumbersome, scalar relations and the solutions are rendered by large scale computations together with time consuming computer codes [15], [16].

The Hexapod manipulator represents a new development in machine tools by utilizing parallel kinematical structures. At the beginning, hexapods were developed, based on the Stewart-Gough platform. More recently, commercial hexapods have been used by many companies in the development of high precision machine tools [17], [18], [19], such as Variax from Giddings & Lewis, Tornado from Hexel Corp. And Geodetic from Geodetic Technology Ltd. Sliding-leg Hexapods with constant lengths have been envisaged, for example, HexaM from Toyoda [20].

Dynamic modelling and analysis of parallel mechanisms is an important part of hexapod design and control. A great deal of work has been done in this area. For example, Fitcher [21] used the Plücker line coordinates for dynamic analysis of parallel manipulators. Sugimoto [22] applied the motor algebra to address the

same problem. Merlet [23] discussed the force transformation between the joint space and the task space, while Do and Yang [24] studied the inverse dynamics based on the Newton-Euler formulation. Geng et al. [25] studied the same problem using Lagrange's formalism. Hashimoto and Kimura [26] applied the parallel computational schemes to the inverse dynamics formulated based on the Newton-Euler method. Fijany and Bejczy [27] utilized a hierarchical graph-based mapping approach to improve the parallel computational schemes.

It should be noted that the previous studies on the dynamics analysis of parallel mechanisms were based on the assumption that leg inertia is negligible. This may be true for Stewart platform-based fight simulator, because the moving platform is much heavier. For machine tools, especially for high-speed machining, the moving platform is lighter and leg of inertia may not be negligeable. Ji [28] first addressed the problem of the effect of leg inertia on the Stewart platform dynamics and provided a dynamic modelling based on the Newton-Euler approach. Recently, Codourey [29] developed a dynamic model including leg inertia for a revolute type of parallel mechanism called Delta. So far, however, no quantitative studies have been made on the effect of leg inertia. This problem is important, because the leg inertia of the Hexapod is compatible to that of the moving platform and dynamics becomes significant when operating at high speeds.

The natural orthogonal complement method has been applied to studying the serial or parallel manipulators and the flexible mechanisms (Angeles and Lee [30], Zanganeh et al. [31], Xi et al. [32]). In this development, the Newton-Euler formulation is used to model the dynamics equations of each individual body, including the moving platform and the legs.

In the present paper, a new recursive matrix method is introduced. It has been proved to reduce the number of equations and computation operations significantly by using a set of matrices for kinematics modelling.

2. Inverse kinematics

A spatial 6-DOF parallel manipulator, which can be existent in several applications including machine tools, is proposed in this paper. Since the pneumatic joints can easily achieve high accuracy and heavy loads, the majority of the 3-DOF or of the 6-DOF parallel mechanisms use the actuated prismatic joints.

The Hexapod under consideration is a six-degree-of-freedom parallel mechanism with constant length sliding legs. As shown in Fig. 1, it is a composed of six guide-ways, six legs, a base and a moving platform. Each leg is connected on one extremity by a universal joint to the guide-way along which the leg slides

and on the other one by a spherical joint to the moving platform to which a tool is attached. The lengths of the legs and their guide-ways are constant.

Symbolically represented by six pairs of spherical joints $A_4, B_4, C_4, D_4, E_4, F_4$ a polygonal moving platform is driven by six sliding legs. Other two polygonal parallel platforms, which are connected by six guide-ways attached at the points $A_0, B_0, C_0, D_0, E_0, F_0$ constitute the fixed base of the manipulator. In what follows we consider that the moving platform is initially located at a *central configuration*, where the moving platform is not rotated with respect to the fixed base and the mass centre G is at an elevation $OG = h$ above the centre of the fixed base.

For the purpose of analysis, a Cartesian coordinate system $Ox_0 y_0 z_0 (T_0)$ we attached to the fixed base with its origin located at the centre O of the fixed platform, the Oz_0 axis perpendicular to the base and the Ox_0 axis pointing to the midpoint line linking the points F_0, A_0. Another coordinate central frame $Gx_G y_G z_G$ could be linked just at the centre G of the moving platform.
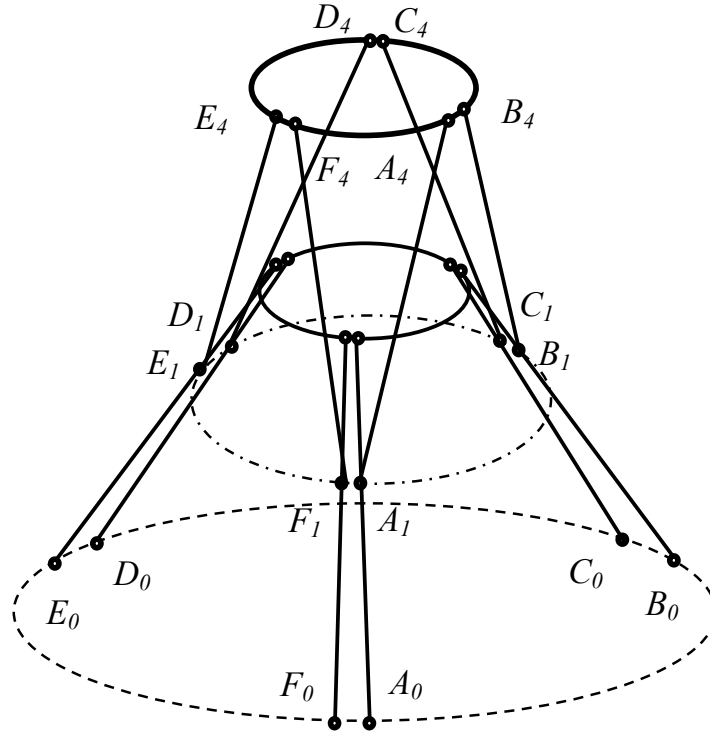

Fig. 1 General scheme of the Hexapod parallel robot

To simplify the graphical image of the kinematical scheme of the mechanism, in what follows the intermediate reference systems has been represented by only two axes, so as is often used in most of robotics papers [1], [5], [11]. The z_k axis is represented, of course, for each component element T_k. It is noted that the relative rotation with angle $\varphi_{k,k-1}$ or the relative translation of the body T_k with the displacement $\lambda_{k,k-1}$ must always be pointed along the direction of the z_k axis.
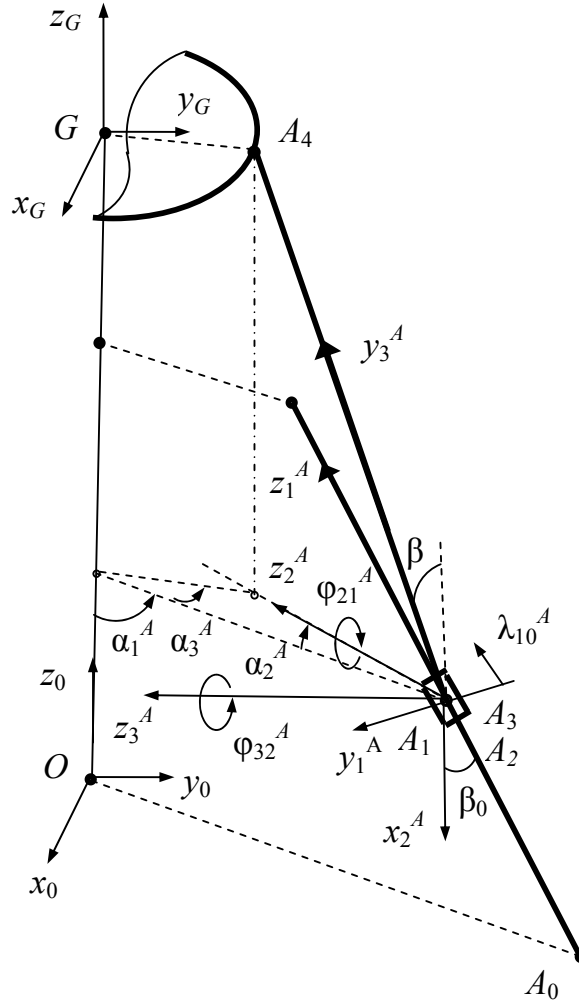

Fig. 2 Kinematical scheme of first leg A of the mechanism

One of these identical active legs (for example leg $A \equiv A_1 A_2 A_3 A_4$) consists of a slider of mass m_1, which effects a translation with the velocity $v_{10}^A = \dot{\lambda}_{10}^A$ and the

acceleration $\gamma_{10}^A = \ddot{\lambda}_{10}^A$, a moving Hooke joint characterized by the mass m_2, the angular velocity $\omega_{21}^A = \dot{\varphi}_{21}^A$ and the angular acceleration $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$, and a leg of constant length l_3, which is connected to the universal joint at the bottom end and a passive spherical joint at the other. This leg has a relative rotation about $A_3 z_3^A$ axis with the angle φ_{32}^A, so that $\omega_{32}^A = \dot{\varphi}_{32}^A$, $\varepsilon_{32}^A = \ddot{\varphi}_{32}^A$ (Fig. 2).

Finally, a ball-joint or a spherical joint is attached to the moving platform, which can by schematised as a polygon. Following notations are used: l_0 radius of the circle associated to the moving platform, $L_p = 2l_0 \sin(\frac{\pi}{3} - \alpha_0)$ long side, $l_p = 2l_0 \sin\alpha_0$ short side, L_0 radius of the circle associated to the fixed base, $L_b = 2L_0 \sin(\frac{\pi}{3} - \alpha_0)$ long side, $l_b = 2L_0 \sin\alpha_0$ short side, $s = 2l_1 = 2(L_b - L_p)/\sqrt{2}$ guide-way length, $\sin\beta_0 = \frac{L_0 - l_0}{l_1}$ inclination of the guide-way, γ guide-way angle, $\sin\beta = 2(\frac{l_0}{l_3})\sin(\frac{\pi}{6} - \alpha_0)$ initial inclination of the leg and $h = l_1 \cos\beta_0 + l_3 \cos\beta$ as initial position of the centre G of moving platform.

At the central configuration, we also consider that the angles of orientation giving the positions of sliders, legs, universal joints and spherical joints are given by

$$\alpha_1^A = \alpha_0, \alpha_1^B = \frac{2\pi}{3} - \alpha_0, \alpha_1^C = \frac{2\pi}{3} + \alpha_0$$

$$\alpha_1^D = -\frac{2\pi}{3} - \alpha_0, \alpha_1^E = -\frac{2\pi}{3} + \alpha_0, \alpha_1^F = -\alpha_0.$$

$$\alpha_2^A = \alpha_2^C = \alpha_2^E = -\frac{\pi}{3} - \alpha_0, \ \alpha_2^B = \alpha_2^D = \alpha_2^F = \frac{\pi}{3} + \alpha_0$$

$$\alpha_3^A = \alpha_3^C = \alpha_3^E = \frac{\pi}{3} - 2\alpha_0, \alpha_3^B = \alpha_3^D = \alpha_3^F = -\frac{\pi}{3} + 2\alpha_0. \tag{1}$$

Assuming that the each leg is connected to the fixed base by the slider and the universal joint such that it cannot rotate about the longitudinal axis, the orientation of the leg A with respect to the fixed base can be described by two Euler angles, namely a rotation angle φ_{21}^A about the $A_2 z_2^A$ axis, followed by another rotation of angle φ_{32}^A about the rotated $A_3 z_3^A$ axis.

Pursuing the first leg A in the $OA_1 A_2 A_3 A_4$ way, we obtain the following matrices of transformation [33]:

$$a_{10} = a_\beta^\circ \theta_3 a_{1\alpha}^A, \ \ a_{21} = a_{21}^\varphi \theta_1 a_{2\alpha}^{AT} a_\beta^{\circ T}, \ \ a_{32} = a_{32}^\varphi a_\beta \theta_1 \theta_2, \tag{2}$$

where

$$a_{i\alpha}^A = \begin{bmatrix} \cos\alpha_i^A & \sin\alpha_i^A & 0 \\ -\sin\alpha_i^A & \cos\alpha_i^A & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (i=1,2,3)$$

$$a_\beta^\circ = \begin{bmatrix} \cos\beta_0 & 0 & -\sin\beta_0 \\ 0 & 1 & 0 \\ \sin\beta_0 & 0 & \cos\beta_0 \end{bmatrix}, \quad a_\beta = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \theta_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \theta_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$a_{k,k-1}^\varphi = \begin{bmatrix} \cos\varphi_{k,k-1}^A & \sin\varphi_{k,k-1}^A & 0 \\ -\sin\varphi_{k,k-1}^A & \cos\varphi_{k,k-1}^A & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad a_{k0} = \prod_{s=1}^{k} a_{k-s+1,k-s}, \quad (k=1,2,3).$$

Analogous relations can be written for other five legs of the mechanism.

Six independent displacements $\lambda_{10}^A, \lambda_{10}^B, \lambda_{10}^C, \lambda_{10}^D, \lambda_{10}^E, \lambda_{10}^F$ of the active links are the input variables that can give the instantaneous position of the mechanism. In the inverse geometric problem, it can be considered that three coordinates x_0^G, y_0^G, z_0^G of mass centre G of the moving platform and others three Euler angles $\alpha_1, \alpha_2, \alpha_3$ of successive rotations about the Gx_G, Gy_G, Gz_G axes gives the position of the mechanism. Since all rotations take place successively by respect to the moving coordinate axes, the resulting rotation matrix is obtained by multiplying three basic rotation matrices:

$$R_1 = R(x,\alpha_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}$$

$$R_2 = R(y,\alpha_2) = \begin{bmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{bmatrix} \tag{4}$$

$$R_3 = R(z,\alpha_3) = \begin{bmatrix} \cos\alpha_3 & \sin\alpha_3 & 0 \\ -\sin\alpha_3 & \cos\alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Then, the general rotation matrix R of the platform from $Ox_0y_0z_0(T_0)$ to $Gx_Gy_Gz_G$ reference system is given by

$$R = R_3 R_2 R_1 \tag{5}$$

It is assumed that the coordinates of the platform's centre G and the and the angles
$\alpha_1, \alpha_2, \alpha_3$, which are expressed by following analytical functions

$$x_0^G = x_0^{G*}(1 - \cos\frac{\pi}{3}t), \, y_0^G = y_0^{G*}(1 - \cos\frac{\pi}{3}t), \, z_0^G = h - z_0^{G*}(1 - \cos\frac{\pi}{3}t) \qquad (6)$$

$$\alpha_l = \alpha_l^*(1 - \cos\frac{\pi}{3}t), \quad (l = 1, 2, 3),$$

can describe the general absolute motion of the moving platform.

The set of 18 variables $\lambda_{10}^A, \varphi_{21}^A, \varphi_{32}^A, \dots, \lambda_{10}^F, \varphi_{21}^F, \varphi_{32}^F$ will be determined by several vector-loop equations established along the branches of the leg-guide-way system, as follows:

$$\vec{r}_{10}^A + \sum_{k=1}^{3} a_{k0}^T \vec{r}_{k+1,k}^A - R^T \vec{r}_G^{A_4} =$$

$$= \vec{r}_{10}^B + \sum_{k=1}^{3} b_{k0}^T \vec{r}_{k+1,k}^B - R^T \vec{r}_G^{B_4} = \qquad (7)$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$= \vec{r}_{10}^F + \sum_{k=1}^{3} f_{k0}^T \vec{r}_{k+1,k}^F - R^T \vec{r}_G^{F_4} = \vec{r}_0^G,$$

where

$$\vec{r}_{10}^A = L_0 a_{1\alpha}^{AT} \vec{u}_1 + (l_1 + \lambda_{10}^A) a_{10}^T \vec{u}_3), \; \vec{r}_{21}^A = \vec{0}, \; \vec{r}_{32}^A = \vec{0}, \; \vec{r}_{43}^A = l_3 \vec{u}_2, \; \vec{r}_G^{A_4} = l_0 a_{1\alpha}^{AT} a_{3\alpha}^{AT} \vec{u}_1$$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \vec{r}_0^G = \begin{bmatrix} x_0^G \\ y_0^G \\ z_0^G \end{bmatrix}$$

$$\tilde{u}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \tilde{u}_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \tilde{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \qquad (8)$$

Knowing the general motion of the platform by the relations (6), the inverse kinematical problem has been developed determining the absolute velocities $\vec{v}_{k0}^A$, $\vec{\omega}_{k0}^A$ and accelerations $\vec{\gamma}_{k0}^A$, $\vec{\varepsilon}_{k0}^A$ of each of the moving bodies.

First, the angular velocities of six legs and the velocities of the joints in terms of the angular velocity of the moving platform and the velocity of its centre G were computed:

$$\vec{\omega}_{60}^{\circ G} = R^T \vec{\omega}_{60}^G = \dot{\alpha}_1 R_1^T \vec{u}_1 + \dot{\alpha}_2 R_1^T R_2^T \vec{u}_2 + \dot{\alpha}_3 R^T \vec{u}_3$$

$$\dot{\vec{r}}_0^G = R^T \vec{v}_{60}^G = [\dot{x}_0^G \quad \dot{y}_0^G \quad \dot{z}_0^G]^T. \qquad (9)$$

The motions of the compounding elements of each leg (for example the leg A) are characterized by following skew-symmetric matrices [34]:

$$\widetilde{\omega}_{k0}^A = a_{k,k-1}\widetilde{\omega}_{k-1,0}^A a_{k,k-1}^T + \omega_{k,k-1}^A \widetilde{u}_3, \ (k = 2, 3), \tag{10}$$

which are *associated* to the absolute angular velocities given by the recursive formula:

$$\vec{\omega}_{k0}^A = a_{k,k-1}\vec{\omega}_{k-1,0}^A + \omega_{k,k-1}^A \vec{u}_3. \tag{11}$$

Following relations give the velocities $\vec{v}_{k0}^A$ of joints A_k:

$$\vec{v}_{k0}^A = a_{k,k-1}\vec{v}_{k-1,0}^A + a_{k,k-1}\widetilde{\omega}_{k-1,0}^A \vec{r}_{k,k-1}^A + v_{k,k-1}^A \vec{u}_3, \ v_{\sigma,\sigma-1}^A = 0 \ (\sigma = 2, 3). \tag{12}$$

Equations of geometrical constraints (7) will be derivated with respect to time in order to obtain the following *matrix conditions of connectivity* established for the characteristic relative velocities of first leg A (for example):

$$v_{10}^A \ \vec{u}_j^T a_{10}^T \vec{u}_3 + \omega_{21}^A l_3 \ \vec{u}_j^T a_{20}^T \widetilde{u}_3 a_{32}^T \vec{u}_2 - \omega_{32}^A l_3 \vec{u}_j^T a_{30}^T \vec{u}_1 = \vec{u}_j^T \dot{\vec{r}}_0^G + \vec{u}_j^T R^T \widetilde{\omega}_{60}^G \vec{r}_G^{A_4}, \tag{13}$$

$$(j = 1, 2, 3)$$

where

$$\widetilde{\omega}_{60}^{\circ G} = R^T \widetilde{\omega}_{60}^G R = \dot{\alpha}_1 R_1^T \widetilde{u}_1 R_1 + \dot{\alpha}_2 R_1^T R_2^T \widetilde{u}_2 R_2 R_1 + \dot{\alpha}_3 R^T \widetilde{u}_3 R \tag{14}$$

denotes the skew-symmetric matrix associated to the absolute angular velocity $\vec{\omega}_{60}^{\circ G}$ of the moving platform [35], [36]. From these equations, we obtain the relative velocities v_{10}^A, ω_{21}^A, ω_{32}^A as functions of angular velocity of the platform and velocity of mass centre G. Derived from (13), the complete Jacobian matrix of the robot is a fundamental element for the analysis of singularity loci and workspace of the robot.

As for the relative accelerations γ_{10}^A, ε_{21}^A, ε_{32}^A of the elements of first leg A of the mechanism, following other *conditions of connectivity* are imposed

$$\gamma_{10}^A \ \vec{u}_j^T a_{10}^T \vec{u}_3 + \varepsilon_{21}^A l_3 \ \vec{u}_j^T a_{20}^T \widetilde{u}_3 a_{32}^T \vec{u}_2 - \varepsilon_{32}^A l_3 \vec{u}_j^T a_{30}^T \vec{u}_1 =$$
$$= \vec{u}_j^T \ddot{\vec{r}}_0^G + \vec{u}_j^T R^T \{\widetilde{\omega}_{60}^G \widetilde{\omega}_{60}^G + \widetilde{\varepsilon}_{60}^G\}\vec{r}_G^{A_4} -$$
$$- l_3 \omega_{21}^A \omega_{21}^A \vec{u}_j^T a_{20}^T \widetilde{u}_3 \widetilde{u}_3 a_{32}^T \vec{u}_2 + l_3 \omega_{32}^A \omega_{32}^A \vec{u}_j^T a_{30}^T \vec{u}_2 +$$
$$+ 2 l_3 \omega_{21}^A \omega_{32}^A \vec{u}_j^T a_{20}^T \widetilde{u}_3 a_{32}^T \vec{u}_1,$$
$$(j = 1, 2, 3), \tag{15}$$

where an useful square matrix is introduced:

$$\widetilde{\omega}_{60}^{\circ G} \widetilde{\omega}_{60}^{\circ G} + \dot{\widetilde{\omega}}_{60}^{\circ G} = R^T (\widetilde{\omega}_{60}^G \widetilde{\omega}_{60}^G + \dot{\widetilde{\omega}}_{60}^G)R =$$
$$= \ddot{\alpha}_1 R_1^T \widetilde{u}_1 R_1 + \ddot{\alpha}_2 R_1^T R_2^T \widetilde{u}_2 R_2 R_1 + \ddot{\alpha}_3 R^T \widetilde{u}_3 R +$$
$$+ \dot{\alpha}_1^2 R_1^T \widetilde{u}_1 \widetilde{u}_1 R_1 + \dot{\alpha}_2^2 R_1^T R_2^T \widetilde{u}_2 \widetilde{u}_2 R_2 R_1 + \dot{\alpha}_3^2 R^T \widetilde{u}_3 \widetilde{u}_3 R +$$
$$+ 2 \dot{\alpha}_1 \dot{\alpha}_2 R_1^T \widetilde{u}_1 R_2^T \widetilde{u}_2 R_2 R_1 + 2 \dot{\alpha}_2 \dot{\alpha}_3 R_1^T R_2^T \widetilde{u}_2 R_3^T \widetilde{u}_3 R + 2 \dot{\alpha}_3 \dot{\alpha}_1 R_1^T \widetilde{u}_1 R_2^T R_3^T \widetilde{u}_3 R. \tag{16}$$

The accelerations $\vec{\gamma}_{k0}^{A}$ of the joints A_k and the angular accelerations $\vec{\varepsilon}_{k0}^{A}$ are expressed by some recurrence relations, founded by derivatives of equations (10), (11) and (12):

$$\vec{\varepsilon}_{k0}^{A} = a_{k,k-1}\vec{\varepsilon}_{k-1,0}^{A} + \vec{\varepsilon}_{k,k-1}^{A} + \omega_{k,k-1}^{A} a_{k,k-1}\widetilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{T}\vec{u}_3$$

$$\widetilde{\omega}_{k0}^{A}\widetilde{\omega}_{k0}^{A} + \widetilde{\varepsilon}_{k0}^{A} = a_{k,k-1}(\widetilde{\omega}_{k-1,0}^{A}\widetilde{\omega}_{k-1,0}^{A} + \widetilde{\varepsilon}_{k-1,0}^{A})a_{k,k-1}^{T} + \widetilde{\omega}_{k,k-1}^{A}\widetilde{\omega}_{k,k-1}^{A} + \widetilde{\varepsilon}_{k,k-1}^{A} +$$
$$+ 2a_{k,k-1}\widetilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{T}\widetilde{\omega}_{k,k-1}^{A}$$

$$\vec{\gamma}_{k0}^{A} = a_{k,k-1}\vec{\gamma}_{k-1,0}^{A} + a_{k,k-1}(\widetilde{\omega}_{k-1,0}^{A}\widetilde{\omega}_{k-1,0}^{A} + \widetilde{\varepsilon}_{k-1,0}^{A})\vec{r}_{k,k-1}^{A} + 2v_{k,k-1}^{A}a_{k,k-1}\widetilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{T}\vec{u}_3 + \gamma_{k,k-1}^{A}\vec{u}_3$$

$$\gamma_{\sigma,\sigma-1}^{A} = 0 \qquad (\sigma = 2, 3). \tag{17}$$

If other five kinematical chains of the manipulator are pursued, analogous relations can be easily obtained.

The relations (13) and (15) represent *the inverse kinematics model* of the Hexapod parallel robot. For simulation purposes let us consider a manipulator, with the following characteristics:

$$\alpha_1^{*} = 0, \ \alpha_2^{*} = 0, \ \alpha_3^{*} = \frac{\pi}{12}, \ \alpha_0 = \frac{\pi}{36}, \ \gamma = \frac{\pi}{4}, \ \Delta t = 3\,\text{s}$$
$$x_0^{G*} = -0.05 \text{ m}, \ y_0^{G*} = 0 \text{ m}, \ z_0^{G*} = -0.05 \text{ m} \tag{18}$$
$$L_0 = OA_0 = 0.6 \text{ m}, \ l_0 = l_4 = GA_4 = 0.3 \text{ m}, \ l_3 = 0.5 \text{ m}.$$

Based on the algorithm derived from above equations, a computer program was developed to solve the inverse kinematics of the Hexapod manipulator, using the MATLAB software. For illustration, it is assumed that for a period of three second the moving platform starts at rest from a central configuration and moves along or rotates about one of three orthogonal directions. A numerical study of the robot kinematics is carried out by computation of the time-history evolution of the displacements $\lambda_{10}^{A}, \lambda_{10}^{B}, \lambda_{10}^{C}, \lambda_{10}^{D}, \lambda_{10}^{E}, \lambda_{10}^{F}$, the velocities $v_{10}^{A}, v_{10}^{B}, v_{10}^{C}, v_{10}^{D}, v_{10}^{E}, v_{10}^{F}$ and the accelerations $\gamma_{10}^{A}, \gamma_{10}^{B}, \gamma_{10}^{C}, \gamma_{10}^{D}, \gamma_{10}^{E}, \gamma_{10}^{F}$ of the six prismatic actuators. Following examples are solved to illustrate the algorithm.
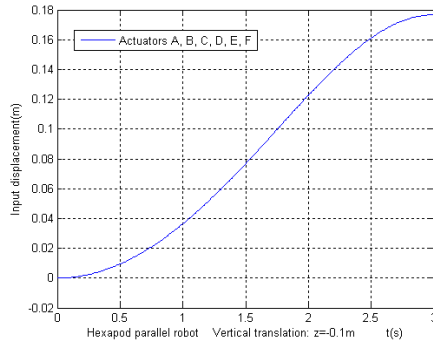

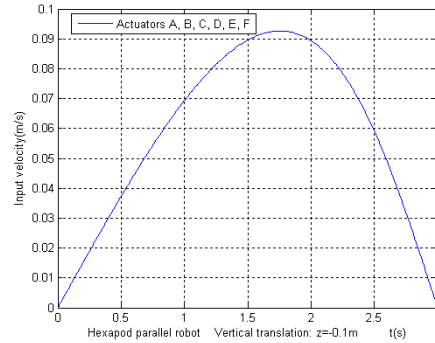Fig. 3 Displacements λ_{10}^{i} of six actuators   Fig. 4 Velocities v_{10}^{i} of six actuators

For the first example, the moving platform moves along the *vertical* z_0 direction with variable acceleration while all the other positional parameters are held equal to zero. As it can be seen from Figs. 3, 4, 5 it is proved to be true that all input displacements, velocities and accelerations are permanently equal to one another.
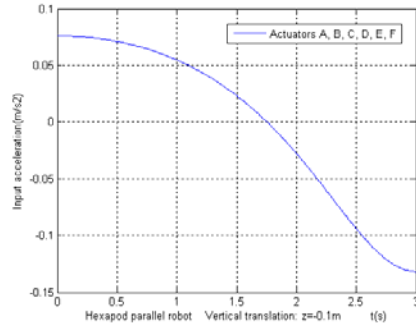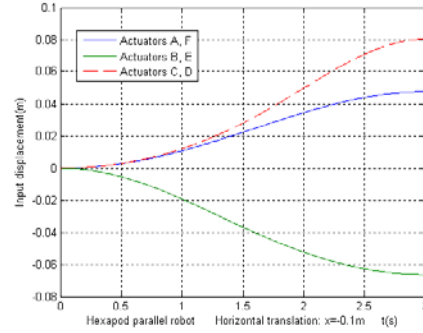

Fig. 5 Accelerations γ_{10}^i of six actuators

Fig. 6 Displacements λ_{10}^i of six actuators

Concerning the comparison in the case when the centre G moves along a *rectilinear trajectory* along the horizontal x_0 axis without rotation of the platform, it is remarked that the distribution of displacement, velocity and acceleration depicted in Figs. 6, 7, 8 is the same, at any instant, for two of six actuators
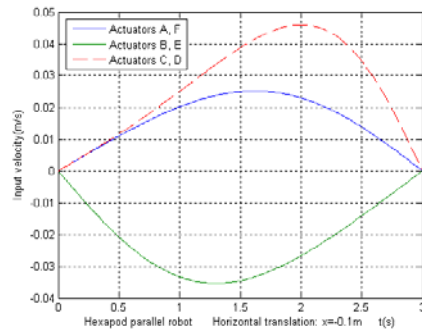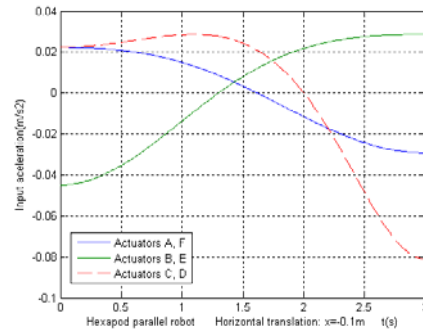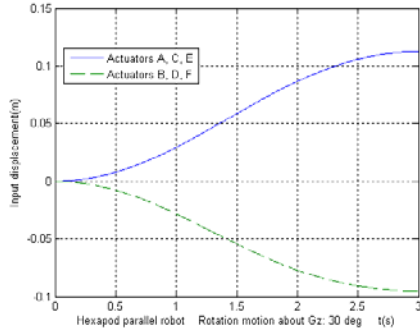

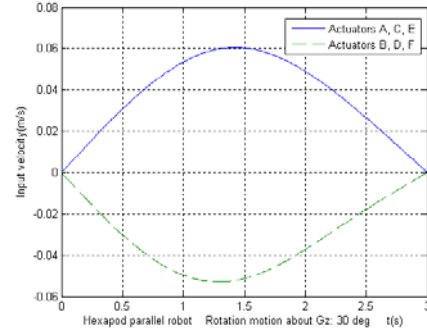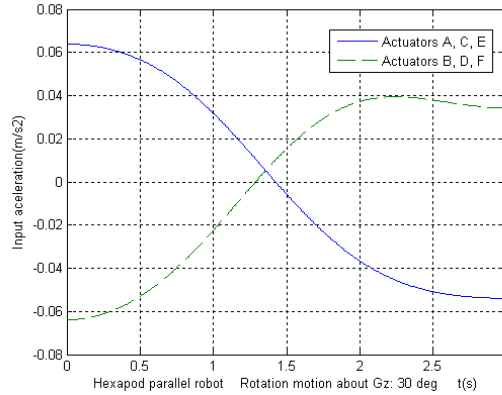Fig. 7 Velocities v_{10}^i of six actuators

Fig. 8 Accelerations γ_{10}^i of six actuators

Fig. 9 Displacements λ_{10}^i of six actuators   Fig. 10 Velocities v_{10}^i of six actuators

For the third example it was considered the *rotation motion* of the moving platform about z_0 direction with a variable angular acceleration $\ddot{\alpha}_3$. The displacements, velocities and accelerations of the six actuators (Figs. 9, 10, 11) are calculated by the program and plotted versus time.

Fig.11 Accelerations γ_{10}^i of six actuators

The simulation through the MATLAB program certifies that one of the major advantages of the current matrix recursive approach is the well structured way to formulate a kinematical model, which leads to a computational efficiency. The proposed method can be applied to various types of complex robots, when the number of components of the mechanism is increased.

3. Conclusions

Within the inverse kinematics analysis some exact matrix relations that give in real-time the position, velocity and acceleration of each element of the parallel robot have been established in present paper. The method described above is quite

available in forward and inverse mechanics of all serial or parallel mechanisms, the platform of which behaves in translation, rotation evolution or general 6-DOF motion.

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