OPTIMAL CONTROL OF A WASTEWATER ACTIVATED SLUDGE SYSTEM

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Procesul de epurare biologică se defineşte drept consumul metabolic al substanţelor organice prezente în apele uzate (industriale sau menajere), propriu unei populaţii mixte de microorganisme (bacterii, ciuperci ş.a.) structurată ca nămol activ. Acesta are loc în sisteme de epurare compuse din bazin de aerare (bioreactor) şi filtru, care separă nămolul activ, recirculat pentru menţinerea unei concentraţii ridicate în sistem, de efluentul epurat. O parte din nămolul separat este eliminat prin purjare, pentru controlul proprietăţilor biomasei, precum şi a concentraţiei de celule moarte din nămolul activ. Indiferent de sistemul de filtrare ales (tangenţial sau normal), acesta se constituie într-un element de nestaţionaritate, care imprimă o dinamică specifică sistemului. Pentru menţinerea calităţii apelor uzate, a fost investigat controlul optimal al sistemului de epurare, considerându-se fracţia de purja drept variabilă de comandă.

The biological wastewater cleaning is defined as the metabolic consumption of organic wastes from water (industrial or municipal) by a mixed population of microorganisms (yeasts, fungi etc) known as activated sludge. The process takes place in systems composed of an aerated tank (bioreactor) and filter, which separates the activated sludge – recycled to maintain its high concentration – from the clean reusable water. A small part of the recycled sludge is removed from the system through a purge, thus controlling the viability of the biomass and the dead cells' concentration. Irrespective of the chosen filtering system (cross-flow or dead-end), it is intrinsically non-stationary, introducing a specific dynamic for the whole system. The optimal control of the activated sludge system previously defined is aimed to search for the right purge fraction which could maintain the same quality of the clean water throughout the operating time.

Keywords: activated sludge system, optimal control, genetic algorithms

Introduction

Nowadays, the activated sludge wastewater (industrial and municipal) treatment became one of the most efficient processes of cleaning and rehabilitating used water, returning it safely into the nature for further reuse. The main drawback of the old technologies is the way the activated sludge is separated from

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the clean water – through gravitational settling, in large basins which occupy useful areas and necessitate impressive investments, even for small wastewater treatment stations. Another disadvantage is the sensitivity to the operating parameters and mainly to the disturbances in the substrates or inflows; sometime their dumping time and amplitude violating the imposed restrictions. The main efforts in the last decades were devoted to improving the adaptability of the process, a better uptake capacity and aeration performance [1]. One of the main improvements of the activated sludge system treatment was the replacement of the settling devices with filters (dead-end or, recently, cross-flow type), which increased the separation rate significantly, diminishing proportionally the investment. Another advantage is the better control of the activated sludge concentration giving a faster response to external perturbations.

Due to the importance of the wastewater activated sludge treatment, its mathematical modeling became a sound research area, aiming to quantify the influence of different operating and system parameters upon the cleaning process. The driving force was the necessity of designing and building even more efficient and cheap wastewater treatment systems, capable of dealing with larger inflows of wastewater, since the domestic and industrial consumption is increasing while the ecological constraints are harder and harder.

Activated sludge wastewater system description

The general scheme of a wastewater treatment facility using activated sludge is depicted in Fig. 1. The main units are the bioreactor, a large aerated basin of tens of thousands of cubic meter, where the activated sludge reacts with the organic wastes from the water transforming them in harmless metabolites, and the filtering system, which separates the clean water from the sludge. The rest is formed of the pipelines and the pump, the purge being also a simple open pipe. The wastewater is mixed with the activated sludge through the compressed air and some mechanical mixing devices. The metabolic processes taking place in the aerated tank need a certain amount of dissolved oxygen, supplied by the compressed air and uniformly distributed due to the mixing. The aeration capacity plays a crucial role in the process of wastewater treatment, but in what follows the assumption of oxygen sufficiency regardless of the operating conditions is considered fulfilled.

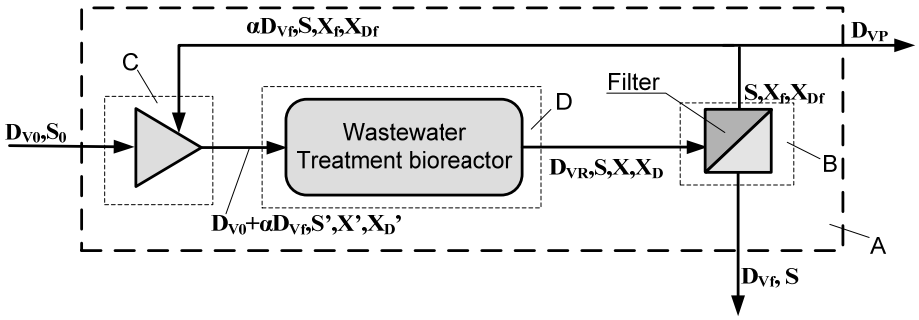

Fig. 1. Activated sludge wastewater treatment system

After spending enough time into the bioreactor (the residence time is such that the exit meets the imposed restrictions), the mixture is separated into the filter, the activated sludge being returned to the bioreactor. This way, a closed circuit is established, facing the associated drawback of inert accumulation (the dead cells) like in any recycling systems. To maintain the inert concentration at an acceptable level, a purge is used, its flow-fraction establishing the amount of the concentration of the dead cells.

The kinetic of the biological process is influenced by the substrate concentration, the presence and the quantity of the essential minerals and oligo-elements, the metabolic product accumulation, the temperature, the concentration of the dissolved oxygen, the ionic equilibrium, and above all, the pH. The main efforts in characterizing these complex dependencies were devoted to capturing them into some simple but rather complete kinetic models, which could then be used to design the wastewater treatment facilities. The most known and widely used, although not very accurate, are the Monod [2] and Haldane [3] models for microorganism's growth and metabolic activities.

Activated sludge recycle permits the increase of the living cells' concentration in the bioreactor and consequently, higher process rates and yields. On the other hand, rising the living cells' concentration beyond a certain level (specific to each microorganism) induces a change in the rheological characteristics of the mixture activated sludge-wastewater, from Newtonian to non-Newtonian (pseudoplastic, mainly), implying the formation of short-cuts or dead zones in the bioreactor, a decrease of the throughput flow and an increase of the filter clogging. More, the dead cells' concentration (seen as an inert for the process) increases continuously, not only occupying valuable reaction space, but contributing, also, to the rheological characteristics' changes [4]. A solution to these problems is to have a purge, after the filtration, throwing away a part of the recycled activated sludge, as depicted in Fig. 1.

The mathematical model of the wastewater treatment process

The mathematical model of the activated sludge wastewater treatment system is of deterministic type, being obtained applying the total and partial mass balances around the key elements of the system, as shown in Fig. 1 (units A, B, C and D). The kinetic of the biological process is described by a Haldane type equation, which takes into account the cells' death. Also, this kinetic is suitable for relatively high living cells' concentrations, encountered when working with recycling systems [5].

Analyzing the envelope A, we observe that there are two outlet flows, D_{VP} and D_{Vf}, and an inlet constant flow, D_{V0}. Since the later outlet flow comes from the filter, whose resistance is time-varying, it will, also, be variable, that is $D_{Vf}(t) = \rho(t) \cdot D_{V0}$, where $\rho(t)$ represents the time evolution of the filtrate ratio with respect to the system inlet flow. The former outlet flow can be, also, related to the system inlet flow, $D_{VP} = \beta \cdot D_{V0}$. Consequently, if β is constant, the volume of the reactor will vary function of the filter outlet, while if $\beta(t) = 1 - \rho(t)$, the bioreactor will operate at constant volume. Taking these into account, the overall mass balance for the system (envelope A) reads:

$$\underbrace{\frac{\mathrm{d}V_R}{\mathrm{d}t}}_{\substack{\text{volume}\\ \text{variation}}} = \underbrace{D_{V0}}_{\text{inlet flow}} - \underbrace{\beta \cdot D_{V0}}_{\text{purge flow}} - \underbrace{\rho(t) \cdot D_{V0}}_{\text{filtrate flow}} \tag{1}$$

Analyzing the equation (1) it becomes obvious that, in order to maintain a constant bioreactor volume, following the filtrate variation, only the inlet flow and the purge fraction, β, can be used. Since the inlet flow depends upon the upstream conditions (being, by itself, a potential perturbation of the system), the only left command is the purge fraction.

Introducing $\upsilon = V_R / V_0$ and $\tau = t/\bar{t}$, where $\bar{t} = V_R / D_{V0}$ is the mean residence time, equation (1) becomes, in its dimensionless form:

$$\frac{\mathrm{d}\upsilon}{\mathrm{d}\tau} = 1 - \beta - \rho(t) \tag{2}$$

which, for the steady-state case simplifies to:

$$\beta = 1 - \rho \tag{3}$$

valid when working at variable pressure drop – but, this mode could damage either the living cells or the filter membrane or both.

Neglecting the cells buildup into the filter, their mass balance around contour *B* gives:

Living cells

$$X_f = X \cdot \frac{1+\alpha \cdot \rho(t)}{\beta + \alpha \cdot \rho(t)} \tag{4}$$

Dead cells

$$X_{Df} = X_D \cdot \frac{1+\alpha \cdot \rho(t)}{\beta + \alpha \cdot \rho(t)} \tag{5}$$

Equations (4) and (5) permit to compute the cells concentration at the reactor inlet, through substitution and, also, the substrate concentration – contour *C*:

Living cells

$$X' = X \cdot \frac{\alpha \cdot \rho(t)}{\beta + \alpha \cdot \rho(t)} \tag{6}$$

Dead cells

$$X'_D = X_D \cdot \frac{\alpha \cdot \rho(t)}{\beta + \alpha \cdot \rho(t)} \tag{7}$$

Substrate

$$S' = \frac{\alpha \cdot \rho(t) \cdot S + S_0}{1+\alpha \cdot \rho(t)} \tag{8}$$

Applying the characteristic equation for the unsteady state perfectly mixed bioreactor (contour *D*), the viable cells concentration reads:

$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}(V \cdot X)}_{\substack{\text{viable cells}\\ \text{accumulation}}} = \underbrace{D_{V0} \cdot \left[1+\alpha \cdot \rho(t)\right] \cdot X'}_{\text{inlet mass flow of the living cells}} - \underbrace{D_{V0} \cdot \left[1+\alpha \cdot \rho(t)\right] \cdot X}_{\text{outlet mass flow of the living cells}} + \underbrace{r_X \cdot V}_{\substack{\text{generation flow}\\ \text{of living cells}}} \tag{9}$$

Introducing the previously defined dimensionless variables, together with the new ones – $\xi = X/X_0$, where $X_0 = Y_{X/S} \cdot S_0$ ($Y_{X/S}$ is the substrate yield in living cells and X_0 is the maximum attainable concentration, if the whole substrate would be transformed into biomass) – and taking into account that $r_X = \mu \cdot X$ equation (9) becomes:

$$\frac{\mathrm{d}\xi}{\mathrm{d}\tau} = \xi' \cdot \frac{1+\alpha \cdot \rho(t)}{\upsilon} - \xi \cdot \left[\frac{2-\beta-\rho(t)\cdot(1-\alpha)}{\upsilon} - \mu \cdot \bar{t}\right] \tag{10}$$

In equation (10), the specific growth rate has an expression of Haldane type:

$$\mu = \frac{\mu_m \cdot S}{K_S + S + \dfrac{S^2}{K_{IS}}} - k_D \tag{11}$$

where:

- μ_m - maximum specific growth rate, , h^{-1}
- K_S - saturation constant, $g \cdot l^{-1}$
- K_{IS} - inhibition through substrate constant, $g \cdot l^{-1}$
- k_D - death rate constant, h^{-1}

Using the dimensionless substrate concentration, $\sigma = S/S_0$, equation (11) becomes:

$$\mu = \frac{\mu_m \cdot \sigma}{K_S^* + \sigma + \frac{\sigma^2}{K_{IS}^*}} - k_D \tag{12}$$

Using (6) and (12) in (10) the final relationship for the dimensionless living cells concentration becomes:

$$\frac{\mathrm{d}\xi}{\mathrm{d}\tau} = \frac{\xi}{\upsilon} \cdot \left\{ \alpha \cdot \rho(t) \cdot \left[1 + \alpha \cdot \rho(t)\right] \cdot \left[1 - \left(1 + \frac{\beta}{\alpha \cdot \rho(t)}\right) \cdot \left(1 + \frac{1 - \beta - \rho(t)}{1 + \alpha \cdot \rho(t)}\right)\right] + \mu \cdot \upsilon \cdot \bar{t} \right\} \tag{13}$$

The final dimensionless dead cells' concentration equation can be obtained analogously:

$$\frac{\mathrm{d}\xi_D}{\mathrm{d}t} = \frac{\xi_D}{\upsilon} \left\{ \alpha \cdot \rho(t) \cdot \left[1 + \alpha \cdot \rho(t)\right] \cdot \left[1 - \left(1 + \frac{\beta}{\alpha \cdot \rho(t)}\right) \cdot \left(1 + \frac{1 - \beta - \rho(t)}{1 + \rho(t) \cdot \alpha}\right)\right] \right\} + \tag{14}$$

$$+ k_D \cdot \xi \cdot \bar{t}$$

Applying the characteristic equation for the unsteady state perfectly mixed bioreactor (contour *D*), the unsteady-state substrate concentration is:

$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}(V \cdot S)}_{\substack{\text{substrate} \\ \text{accumulation}}} = \underbrace{D_{V0} \cdot \left[1 + \alpha \cdot \rho(t)\right] \cdot S'}_{\text{substrate inlet flow}} - \underbrace{D_{V0} \cdot \left[1 + \alpha \cdot \rho(t)\right] \cdot S}_{\text{substrate outlet flowor}} - \underbrace{r_S \cdot V}_{\substack{\text{substrate} \\ \text{consumption} \\ \text{flow}}} \tag{15}$$

In equation (15), the substrate consumption rate, $r_S = \frac{r_X}{Y_{X/S}} + m \cdot X$, takes into consideration the consumption for maintenance, where *m* is the maintenance rate constant. Written for dimensionless concentrations, the substrate mass balance reads:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{\sigma \cdot \left[\beta + \rho(t) - 2\right]}{\upsilon} + \frac{1}{\upsilon} - \left(\mu + m \cdot Y_{X/S}\right) \cdot \xi \cdot \bar{t} \tag{16}$$

To complete the mathematical model, the well known filtering equation, written for constant pressure drop is considered:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = D_V(t) = A_f^2 \cdot \frac{\Delta p^{1-s}}{r_1 \cdot \eta \cdot \varepsilon_s \cdot V_f} \tag{17}$$

The filtrate volume, V_f, collected up to the current time, can be computed using the mean value of the flow, integrating (17):

$$\bar{D}_V(t) = A_f\sqrt{\frac{K_1}{t}} \quad \Rightarrow \quad V_f = \bar{D}_V(t)\cdot t = A_f\sqrt{\frac{K_1}{t}}\cdot t = A_f\sqrt{\frac{2\cdot t\cdot \Delta p^{1-s}}{r_1\cdot\eta\cdot\varepsilon_s}} \tag{18}$$

Using (18), the final correlation for the mean flow becomes:

$$D_V(\tau) = A_f\sqrt{\frac{\Delta p^{1-s}}{2\cdot r_1\cdot\eta\cdot\varepsilon_s\cdot\tau\cdot\bar{t}}} \tag{19}$$

Evidently, at $\tau = 0$ $D_V\big|_{\tau=0} = (1-\beta)\cdot D_{V0}$. Now, a relationship for $\rho(t)$ is at hand:

$$\rho(\tau) = \frac{A_f}{D_{V0}}\sqrt{\frac{\Delta p^{1-s}}{2\cdot r_1\cdot\eta\cdot\varepsilon_s\cdot\tau\cdot\bar{t}}} = \frac{K_f}{\sqrt{\tau}} \tag{20}$$

If the filter is of the cross-flow type, a better equation to be used is:

$$\rho(\tau) = \rho\big|_{\tau=0}\cdot\exp(-K_f\cdot\tau) \tag{21}$$

Results and discussions

As mentioned previously, the control variable was chosen the purge fraction, since both recirculation fraction, dependent upon the inlet flow, constant for this case but which could be seen as perturbation also, and the filter constant are design parameters rather than operating ones.

The optimization was done with Genetic Algorithm, the entire procedure, together with the whole methodology being adapted from [6]. The only difference is that, here, the control variable is the purge fraction, assumed constant for each time interval of the operating period.

Three runs were done, taking β as control variable and allowing only three value for the couple (α, K_f), meaning $(0.2975, 0.1275)$, $(0.35, 0.15)$ and $(0.4025, 0.1725)$, the results being condensed in Table 1. The performance index to be minimized is sum of two dimensionless concentrations: the substrate outlet's and the dead cells':

$$I^* = (\sigma + \xi_D)\big|_{\tau=\tau_{\text{final}}} \tag{22}$$

For each of these runs the same "reference state" was used, as being the performance index obtained for an optimal fixed value of the purge fraction, $\beta = 0.00575$. Details regarding the procedure used to find this optimal value are given in [7].

Looking at the results presented in Table 1, it should be noticed that the performance indexes for an optimum profile of the purge fraction are always bet-

ter (they have lower values) than when the same operating parameter was kept constant (see columns 10 and 11). This shows that choosing β as control variable was a good option. More, the final volumes in all runs were significantly lower than for the case of constant purge fraction, which is a very good result, keeping in mind that the bioreactor volume is very important, not only from the technological point of view, but also from an economic one, since it greatly influences the total investment. Another advantage of using β as control variable is the lower values obtained for the dead cells' concentration, which means that the bioreactor's volume is better used.

Table 1.

Final results of the run tests when considering β as control variable (star marked variables) compared with those obtained keeping β constant, at the optimal value [7]

α	K_f	ξ_f^*	ξ_f	$\xi_{D,f}^*$	$\xi_{D,f}$	σ_f^*	σ_f	υ_f	υ_f^*	I^*	I
0	1	2	3	4	5	6	7	8	9	10	11
0.2975	0.1275	13.21	18.96	2.88	4.21	0.091	0.064	2.79	2.39	0.2226	0.2562
0.35	0.15	12.24	18.23	2.67	4.05	0.091	0.063	3.03	2.59	0.2126	0.2476
0.4025	0.1725	11.51	17.56	2.51	3.91	0.090	0.062	3.25	2.79	0.2044	0.2400

Quite unexpectedly, the final values for the living cells' concentration are lower that those corresponding to the constant β, the performance of the reactor deteriorating accordingly, since the substrate concentration in effluent increased.

Analyzing the performance index (22), it becomes quite clear that the optimization algorithm is focusing especially on the task of minimizing the dead cells' concentration, since the outlet substrate concentration diminishes as a result of the biological process anyway. So, the algorithm will search for the optimal purge fraction profile ensuring a minimum outlet dead cells' concentration.

To overcome this drawback, a better objective function should be used, ensuring a better distribution of the optimization efforts:

$$I^* = \omega \cdot \sqrt{\left(\sigma\big|_{\tau=\tau_{\text{final}}} - \sigma_{\text{imposed}}\right)^2} + \xi_D\big|_{\tau=\tau_{\text{final}}} \tag{23}$$

In equation (23), ω is a penalty value, chosen such that the product $\omega \cdot \sqrt{\left(\sigma\big|_{\tau=\tau_{\text{final}}} - \left(\sigma\big|_{\tau=\tau_{\text{final}}}\right)_{\text{imposed}}\right)^2}$ is of the same order of magnitude as $\xi_D\big|_{\tau=\tau_{\text{final}}}$. This way, the major role of the wastewater activated sludge treatment system is reinforced.

Now, the optimal control problem translates to finding an optimal profile of the purge fraction, $\beta(t)$, which can keep the performance of the bioreactor as close as possible to the target, maintaining, in the same way, the dead cells' con-

centration as low as possible, but this time without sacrificing the main task of the treatment system.

The same run tests were done and the new results are presented in Table 2, for $\omega = 0.15$ and $\left(\sigma\big|_{\tau=\tau_{\text{final}}}\right)_{\text{imposed}} = 0.02$; the distance between the target and the actual treatment system is given by $\left|\sigma^{+}_{\text{final}} - \left(\sigma_{\text{final}}\right)_{\text{imposed}}\right|$.

Analyzing the results presented in Table 2, one observes that wastewater activated sludge treatment system performs well, in terms of the targeted outlet substrate concentration – the distance of the actual output being remarkably close.

This achievement was possible increasing the living cells' concentration and the final volume of the bioreactor, in order to guarantee a larger mass of living cells consuming the same amount of incoming substrate (no attempt was made to simulate the behavior of such systems to some known or random perturbations of the inlet flow or substrate concentrations). This, at the expense of increasing the dead cells' concentration and the investment costs.

Table 2.

The results obtained using (23) as objective function and β as control variable

α	K_f	ξ^{+}_{f}	$\xi^{+}_{D,f}$	$\left\|\sigma^{+}_{f} - \left(\sigma_f\right)_{\text{imposed}}\right\|$	υ^{+}_{f}	I^{+}
0.2975	0.1275	21.3255	4.7761	0.0007	3.1900	0.1313
0.35	0.15	20.3186	4.5204	0.0002	3.3897	0.1141
0.4025	0.1725	19.6223	4.4028	0.0001	3.6302	0.1029

Comparing the results from Table 1 and Table 2, it becomes obvious that a thorough economic analysis should be done, and the optimal control using an economic objective function should be envisaged, in order to obtain the best system.

The results for $\alpha = 0.35$ and $K_f = 0.15$ associated with Table 2 are depicted in Fig. 2, where the time variation of the state and operating parameters are presented, together with the profile of the command variable, the purge fraction β. The optimal policy decreases towards the minimum permissive value, after which it increases towards the maximum admissible one. It should be mentioned that during this passage, substrate consumption curve shows an inflexion point.

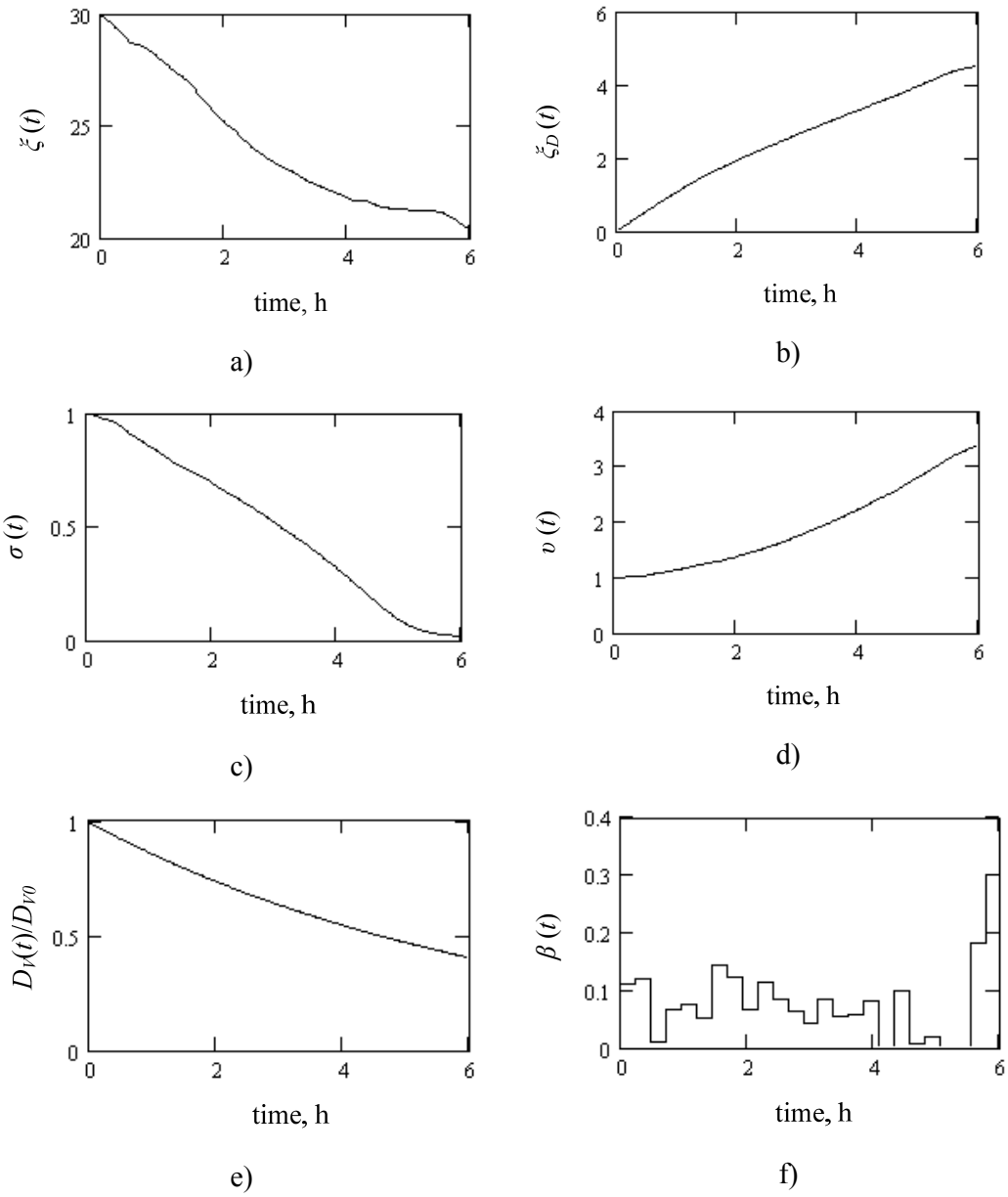

Fig. 2. Time dependency of the state and operating variables for I^+ and $\alpha = 0.35$, $K_f = 0.15$

Conclusions

The mathematical model of the wastewater activated sludge treatment system was built up, using a Haldane-type kinetic for the microorganisms' growth and taking into consideration the cells' death. The mathematical model was used to simulate and, then, optimally control a recycle system composed of a mixer, a bioreactor and a tangential filter.

In a companion study, the influence of the main operating and design parameters (recycle fraction, α, purge fraction, β, and the filter constant, K_f) was investigated. A complete factorial experiment with three levels was used, proving that the concentrations of living and dead cells depend in descending order upon α than β and, finally, K_f, while the final substrate concentration (giving, in fact, the scope of the treatment process) depends, mainly, upon K_f. Another state variable whose value depends mainly upon K_f is the final volume of the bioreactor (taking into account the decrease of the filter throughput flow, an increase of the volume in the bioreactor is normal). These findings shows that the process is more sensitive to α and K_f, and less sensitive to β.

Despite this conclusion, the natural choice is to pick-up β as control variable, since it is the only which can be voluntarily modified and has a global influence upon the whole system, apart from the inlet flow; but, this one is an input variable, which, under various circumstances, can be even a perturbation.

Some preliminary runs showed that there is an optimal value of β, constant in time, for frozen α and K_f (seen as design variables) minimizing some performance indexes like those defined in equations (22) and (23) or alike.

But the real optimal behavior of the aforementioned system is for optimal time profile of the purge fraction, which really minimizes indexes (22) and (23) for the whole operating period.

The derived optimal control are practically implemented by means of a computer control system of the wastewater activated sludge treatment plant, any manual implementation of such complex control function is excluded.

A key aspect, which should be considered in the further development, is how the natural restriction of a finite volume will influence the optimal behavior of the system, including, here, the optimal profile of the command variable, the purge fraction.

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